**Part E**

At steady state:

|  |  |
| --- | --- |
|  |  |

For boundary condition we use Forward approximation for first derivative for Neumann we get = 0 and

We solve for C choosing 6 different meshes: N=[4 40 100 400 4000 40000]

Comparing the graph of the analytical solution with the numerical solutions for different meshes we get:

Chart, line chart

Description automatically generated Chart

Description automatically generated

Chart

Description autonmatically generated with medium confidence Graphical user interface, chart

Description automatically generated with medium confidence

Chart

Description automatically generated with medium confidence Chart

Description automatically generated

The errors L1,L2 and L inf are found using the formulas of the slides. We ge the following figure:

Graphical user interface, chart

Description automatically generated

We calculated the solution for several meshes to be sure to be in the limit of asymptotic convergence. This error must converge to zero with the refinement of the mesh

We get the following values:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| L1 | 0.78125 | 0.078125 | 0.03125 | 0.007813 | 0.000781 | 7.81E-05 |
| L2 | 0.956832 | 0.090773 | 0.036174 | 0.009027 | 0.000902 | 9.02E-05 |
| L3 | 1.5625 | 0.15625 | 0.0625 | 0.015625 | 0.001562 | 0.000156 |

We have a convergence order of E-05. The graphs also show that for a small N the numerical approximation is not very accurate using forward differences.

**Part F**

At steady state:

|  |  |
| --- | --- |
|  |  |

Using Forward approximation for first derivative for Neumann boundary condition, We solve for C choosing 6 different meshes: N=[4 40 100 400 4000 40000]

Graphical user interface, application

Description automatically generated

Graphical user interface, chart

Description automatically generated

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| L1 | 0.221726 | 0.00204 | 0.00032 | 1.97E-05 | 1.96E-07 | 1.95E-09 |
| L2 | 0.322345 | 0.002988 | 0.000463 | 2.81E-05 | 2.77E-07 | 2.76E-09 |
| L3 | 0.654762 | 0.01104 | 0.002053 | 0.000155 | 2.00E-06 | 2.45E-08 |

We have a convergence order of E-09

Central difference method is equivalent to the average of forward and backward difference method when the data points are equally spaced. This method gives a truncation error of second order which provides more accuracy in approximation of the first derivative

**Part B**

**Euler implicit:**

+

|  |  |
| --- | --- |
|  |  |

For boundary condition , we use gear for approximation:

|  |  |
| --- | --- |
| i = 1 |  |
| i = 2 to N |  |
| i = N+1 | = Ce |

|  |  |
| --- | --- |
| **Initial Condition**  = C(r,0)=0 | |
| **Boundary Condition** | |
| Dirichlet |  |
| Neumann (gear approximation) |  |

B=

A= ,

C=

=